AIAA Journal

VOLUME 8 MARCH 1970 NUMBER 3

Panel Flutter: A Review of the Aeroelastic Stability of Plates and Shells

E. H. DOWELL
Princeton University, Princeton, N. J.

Nomenclature

a	= plate length
b	= plate width
D	= plate stiffness
E	= modulus of elasticity
g	= structural damping, $[1 + ig]$ type unless otherwise noted
H	= maximum height rise of constant curvature plate
h	= plate thickness
K	= $\omega(\rho_m h a^4/D)^{1/2}$
M	= Mach number
$N_x^{(a)},N_y^{(a)}$	= applied in-plane loads
Δp	= static pressure differential
P	= $\Delta p a^4/Dh$
q	= $\rho U^2/2$ dynamic pressure
R_x,R_y	= $N_x^{(a)}a^2/D, N_y^{(a)}a^2/D$
t	= time
U	= mean velocity over panel
w	= plate deflection
β	$\equiv (M^2 - 1)^{1/2}$
Γ_x,Γ_y	= $(8H/h)$, $(8H/h)$ $(a/b)^2$, nondimensional curvatures
λ^*	= $2qa^3/D$
λ	= λ^*/M or $\lambda^*/(M^2 - 1)^{1/2}$
μ	= $\rho a/\rho_m h$
δ	= boundary-layer thickness
ρ	= air density
ρ_m	= plate density
τ	= $t(D/\rho_m h a^4)^{1/2}$
ω	= frequency
ξ	$\equiv x/a$
η	$\equiv y/b$

Subscripts

d	= dynamic
f	= flutter
p	= peak; $\xi = 0.75$, $\eta = 0.5$ unless otherwise noted

Introduction

IN this short review an attempt is made to assess the state-of-the-art with regard to panel flutter and, equally important, indicate the areas where fruitful work may be done to improve the state-of-the-art. The review is nonmathematical in the sense that no mathematical analysis is given; on the other hand, the general nature of the various mathematical theories is discussed and thus some knowledge of the usual methods of mathematical physics is assumed. With regard to the experimental aspects, somewhat more space is given to experimental techniques and results than would be justified on the basis of their presence in the literature. However, the meager quantity of published experimental work reflects the difficulty of carrying out meaningful experiments as compared with theoretical work; it does not indicate the relative importance of the two parts of the problem. Indeed the most fruitful work has combined both theory and experiment. For a recent and comprehensive review of the theory complementary to the present survey, the reader may refer to the author's previous article in Fluid-Solid Interaction.[1] Previous surveys on the subject have been written by Fung[2] and Johns.[3,4]

The discussion begins with the physical and experimental aspects of the subject. An outline of the theory follows. The remainder of the discussion is broken up along conventional lines with the various plate or shell geometries being treated separately. It begins with the simplest geometry, flat, unloaded plate, and proceeds to the most complicated, the cylindrical shell, for which any substantial knowledge exists. A conscientious attempt is made to differentiate between what is known beyond a reasonable doubt, what is surmised or guessed, and what is unknown.

No attempt is made to cover all the variations in structural geometry or construction which have been studied and may

Earl H. Dowell is presently Associate Professor, Department of Aerospace and Mechanical Sciences, Princeton University. He received his B.S. from the University of Illinois (1959) and S.M. (1961) and Sc.D. (1964) from the Massachusetts Institute of Technology. He has held positions as Research Engineer with The Boeing Company (1962–63) and M.I.T. (1964) and was an Assistant Professor at M.I.T. (1964–65) under the Ford Foundation post-doctoral program. He has been with Princeton University since 1965. He has published papers in the areas of aeroelasticity, unsteady aerodynamics, acoustics, structural dynamics, and hydrodynamic stability. He is a member of the AIAA.

Received July 1, 1967; revision received December 12, 1969. The author wishes to acknowledge many useful discussions of panel flutter with J. Dugundji of the Massachusetts Institute of Technology and H. M. Voss of The Boeing Company. He would also like to acknowledge his indebtedness to the many investigators who have contributed to this problem. This work was supported by NASA Grant NGR 31-001-059.

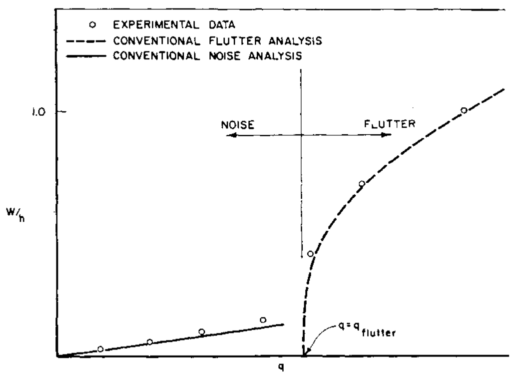

Fig. 1 Schematic of plate response.

be of practical interest. For example, orthotropicity, angle of yaw, and multibay effects are omitted. In the author's experience, while such effects may need to be accounted for in specific applications, they can be taken care of with a moderate amount of effort if the basic theoretical and/or experimental methods are adequate.

Finally, and by way of a disclaimer, any review inevitably reflects to a greater or lesser degree the experience and prejudices of the author. It is hoped that the present review accurately reflects the former and minimizes the latter.

1. Experiment

1.1 Physical Nature of the Problem

Let us begin by considering a wind-tunnel experiment (see, for example, Ref. 5) which has as its purpose the determination of the flutter boundary of a flat, rectangular, isotropic plate at supersonic speeds. Normally the test would begin with the airflow over the panel established at some fixed Mach number, stagnation pressure, and temperature. The flutter boundary is sought by increasing the stagnation pressure (and hence dynamic pressure and density which are proportional to stagnation pressure) with Mach number and stagnation temperature held fixed. Below the flutter boundary, random oscillations of the panel are observed with predominant frequency components near the lower panel natural frequencies; the panel is responding to pressure fluctuations in the turbulent boundary layer. The amplitudes of the oscillations are some small fraction of the panel thickness. As the flutter boundary is exceeded at some critical stagnation pressure or dynamic pressure the oscillation acquires a nearly sinusoidal character with an amplitude which grows to on the order of a plate thickness, when the stagnation pressure or dynamic pressure is increased from 25–100% beyond the value at which flutter begins. See Fig. 1 for a schematic of this behavior.

1.1.1 Identification of onset of flutter

Actually the point at which flutter begins is more a matter of definition than some point which can be determined with great precision. Typically the onset of flutter can be specified within about 10% of the dynamic pressure. There has been some effort to improve matters with regard to a precise experimental determination of the flutter boundary via admittance techniques and the concept of a linear plate impedance.[6] With this technique an external oscillatory force is applied to the plate and the flutter boundary is inferred by the vanishing of the force required to sustain the plate oscillation. The physical attachment providing the force has the advantage of providing a safety device which prevents the destruction of the plate due to flutter; it has the disadvantage of not permitting an investigation of the post-flutter (nonlinear) regime since the force attachment modifies the panel characteristics in the nonlinear regime. Whether the technique is useful or not depends upon the nature of the flutter anticipated, catastrophic failure or fatigue failure. If the flutter or the plate is of the milder, fatigue type the uncertainty of the definition of the flutter boundary is inherent in the physical nature of the problem and a very precise definition of the flutter boundary is neither meaningful nor useful.

1.1.2 Character of post-flutter oscillation

The post-flutter behavior is largely dominated by the nonlinearities of the system. Most prominent of these is the nonlinear structural coupling between bending and stretching of the plate. As the plate bends it also stretches thereby inducing a tension in the plate. The post-flutter oscillation represents a balance between the (unstable) linear plate and fluid forces and this tension which increases the effective plate stiffness. Recognizing this balance one can obtain qualitative estimates of the flutter amplitude by order of magnitude considerations.[7]

1.1.3 Failure due to flutter

Not a great deal of study has been directed toward failure mechanisms. However, at least two are readily identifiable and have occurred in practice. If the stress amplitude due to flutter exceeds the yield stress of the plate material then catastrophic or rapid failure occurs; on the other hand, if the stress is relatively small then fatigue or long-time failure may occur. From a knowledge of stress amplitude and frequency of the oscillation, an estimate of the fatigue life may be made.

1.2 Experimental Methods

1.2.1 Other flutter testing techniques

The determination of the flutter boundary by raising the tunnel stagnation pressure is not the only one which may be employed. For example, Lock and Fung[8] have changed Mach number at constant stagnation pressure. On the other hand, the Langley group in their many tests of the flutter of buckled plates (see e.g., Ref. 9) have usually penetrated the flutter regime by heating the panel at constant stagnation pressure to induce compressive thermal stresses which decrease plate stiffness. The latter investigation employed a blowdown wind tunnel where changing stagnation pressure in precise steps is impractical. Although the use of a blowdown tunnel is feasible, it imposes severe requirements with regard to the determination and control of the plate environment, particularly with regard to thermal stresses and static pressure loading. Generally the use of a continuous flow wind tunnel is to be recommended so that at each value of stagnation pressure and Mach number, the temperature and static pressure loading may be brought to the desired equilibrium values.

Finally, it should be emphasized that before experimental techniques were refined and all the pertinent variables were known, many anomalous experimental results were found due to imperfect experimental techniques. It does not seem desirable to review these here, however.

1.2.2 Prewind tunnel preparation

Models: The success of a wind-tunnel test is largely determined before the test specimen is placed in the tunnel. Therefore, consider the pretunnel work which must be done.[5] The first task, and often the most difficult, is the construction of the plate models and their boundary support. Because of the dynamic pressure ranges available in most supersonic wind tunnels the plate thickness/length ratio must be quite small, $10^{-2} - 10^{-3}$, in order to obtain flutter data. This becomes a particular problem at high supersonic Mach number where the dynamic pressure required for flutter increases.

The fabrication of these thin plates and the mounting on their boundary supports without inducing significant stresses is a major concern. At lower Mach numbers where thicker plates may be used this is a lesser problem. Experience has shown that bench vibration tests to determine panel natural frequencies and modes are illuminating indicators of the quality of model construction. If a good job has been done the measured frequencies and modes will agree closely with their theoretical counterparts.

Pressure differential and thermal stresses: The next step is to determine the sensitivity of the models to static pressure loadings and thermal stresses due to a temperature differential between model and support. These effects, again for the thinner plates, may require considerable effort to control and measure in the wind tunnel. Hence, it is essential that the sensitivity of the model to these factors be determined early in the experimental program by bench vibration tests. The sensitivity can also be estimated on theoretical ground but vibration tests are generally more informative particularly with respect to assessing in-plane edge support conditions and not all that difficult to perform.

Cavity effect: Another concern may be the dynamics of the air in an enclosed cavity beneath the plate. If the cavity depth is on the order of or greater than the plate length or width, the air in the cavity will not play an important role. However, for shallow cavity depths, which are sometimes dictated by practical considerations such as wind-tunnel blockage, vibration tests with and without the cavity will be desirable as well. Actually, the bench vibration test will show a much larger cavity effect than will be present in the wind tunnel since the ambient density and pressure are much larger. The cavity effect is not always negligible in the tunnel, however, The available evidence suggests that theory can predict the experimental data for the bench vibration test and properly scale the cavity effect to the wind-tunnel condition. Hence, unless an unusual cavity is used, cavity vibration tests are less essential than those cited above.

Tunnel and instrumentation selection: After the pretunnel program has been completed it remains to mount the panel and its support in the wind tunnel and to install the appropriate instrumentation with supporting equipment. One might also say that it remains to choose an appropriate wind tunnel; however, in practice the choice is often rather limited. Generally speaking, one wants a wind tunnel with the appropriate Mach number range (generally supersonic), largest possible size and dynamic pressure range, and with good control over tunnel temperature.

With regard to tunnel mounting, two techniques have been used: 1) wall mount and 2) splitter plate. See Fig. 2a and 2b. Generally, the former is more desirable from the point of view of easier access though practical considerations may preclude its use in some tunnels.

The choice of instrumentation is usually made so that the temperature of model and support, pressure differential across model, plate deflection and frequency can be measured. This naturally leads to the use of thermocouples, pressure (0.01–1.0 psi) transducers and strain gauges. Capacitance or inductance deflection transducers are often used though other types have been employed successfully. The general ground rule is to select lightweight, if possible noncontacting, transducers which will not alter the dynamic characteristics of the plate nor disturb the aerodynamic flow.

Once the model is installed most of the work is done and the test is carried out as outlined in the previous section. The major difference in the success of various experimental investigations has been in the thoroughness of the pretunnel program.

1.3 Experimental Results

We shall defer a discussion of results until they may be compared to theory. It will also be noted that no discussion was given with regard to flight testing. Many of the experimental techniques will, of course, be similar to those of wind-tunnel testing. In particular, for pretunnel in the previous discussion one may insert preflight test. Most of the results of flight testing are classified and hence, will not be discused in this review.

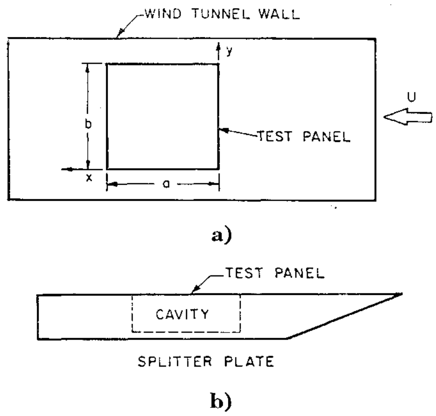

Fig. 2 Wall mount splitter plate.

2. Theory

2.1 General Discussion

Although there has been a voluminous theoretical literature on this problem over the past decade, most analyses can be placed in one of four categories based on the structural and aerodynamic theories employed: 1) linear structural theory; quasi-steady aerodynamic theory (see Refs. 10 and 11 plus others too numerous to mention); 2) linear structural theory; full linearized (inviscid, potential) aerodynamic theory (e.g., Refs. 5 and 12); 3) nonlinear structural theory; quasi-steady aerodynamic theory (e.g., Refs. 13–18); 4) nonlinear structural theory; full linearized (inviscid, potential aerodynamic theory Refs. 19 and 20).

Actually, only for the flat plate and shallow curved plates have all four of the above been carried through to the point of obtaining quantitative results. For other geometries 2 and 4, particularly, have not been studied; there would appear to be no reason why they should not be in the near future, however.

Of these four categories, 1 comprises the great bulk of the literature for the very good reason that it is the simplest. Unfortunately, a type 1 analysis has two major weaknesses: a) it does not account for structural nonlinearities; hence, it can only determine the flutter boundary and can give no information about the flutter oscillation itself; b) the use of quasi-steady aerodynamics neglects the three-dimensionality and unsteadiness (or memory) of the flow; hence, it cannot be used too close to a Mach number of one. However, it is in the vicinity of $M = 1$ that flutter is most likely to occur.

A type 2 analysis is intended to remedy b. It does so in large measure though some have suggested the effect of the boundary layer and transonic flow nonlinearities must also be accounted for near $M = 1$. This is true under some conditions as will be discussed subsequently. No rational flutter analysis has been made which accounts for transonic nonlinearities to date; only quite recently has the boundary-layer effect been included in a realistic way. See Sec. 4 for further discussion of these points. Type 2 still has weakness a.

A type 3 analysis remedies a but still possesses weakness b.

A type 4 analysis remedies both a and b and represents the most advanced state-of-the-art (though again see Sec. 4 for discussion of the boundary-layer effect).

2.2 Type 4 Analysis

Type 4 contains as special cases 1, 2, and 3 by deleting appropriate terms in the governing equations. Therefore, for brevity we will be content with a discussion of 4 and subsequently indicate the simplifications of 1, 2, and 3. The essence of the analysis[18,19] may be indicated as follows: expand the structural deformation in a series of natural, or at least complete, modes; determine the aerodynamic forces for the given modal deformtion, e.g., using the techniques of the transform calculus; using the two preceding steps in the equations of motion apply Galerkin's method to arrive at a set of ordinary, nonlinear, integral-differential equations in time for the modal amplitudes.

By omitting the nonlinear terms (due to structural nonlinearities) and the integral terms (which account for the memory and three dimensionality of the flow) one has a type 1 analysis. By omitting the nonlinear terms only one has a type 2 analysis and by omitting the integral terms only one has a type 3 analysis.

Whatever the level of approximation, the set of equations may be solved for the plate motion by a numerical integration with respect of time. If the nonlinear terms are omitted the oscillating motion will either decay or grow (exponentially) depending on whether the plate is stable or unstable (flutters). If the nonlinear terms are present the unstable motion will terminate in a finite amplitude or limit cycle oscillation.

2.3 Other Theoretical Methods

Very briefly the aforementioned describes the essentials of much of the theoretical literature. There are many individual variations from investigator to investigator, of course. Some of these are discussed here and contrasted with the aforementioned analysis.

First of all, if the problem is studied within the linear formulation, historically the approach to an investigation of stability has been from the point of view of an eigenvalue problem. In the author's experience with both methods the numerical integration or simulation approach has proven more economical and more informative.

Secondly, various methods have been used to obtain the full linearized aerodynamic forces in the literature. Most notable and distinct from the transform approach of Ref. 20 is the box method (Ref. 12), which is essentially a finite difference solution. While the box method is more versatile (it may be used for lifting surface flutter) it is generally less efficient than the use of Fourier and Laplace transforms for panel flutter.

Lastly, a number of approaches have been used in representing the plate deflection and deriving the equations of motion. Three alternative approaches to a modal expansion combined with Galerkin's method are: 1) finite difference representations, 2) finite element representations, and 3) separation of variables or so-called exact solutions. With regard to 1, one is faced with the usual compromise of the Galerkin expansion requiring fewer degrees of freedom than the finite-difference approach (the number of modes being much smaller than the number of elements for given accuracy) but the latter being much easier to set up for digital computation and, in principle, being more versatile. 2) Olson[21] has performed a type 1 analysis using the finite element approach and successfully reproduced results previously obtained by other methods. The advantage of the finite element method over the finite difference approach is that since each element is constrained to behave in a realistic physical manner one should be able to use a smaller number of elements (degrees of freedom) than in the finite difference approach. The price to be paid is in the identification of a suitable element. In particular, whether one can usefully extend this approach to the type 2–4 analyses is an open question. Of course, the finite difference method is capable of such extension although the requisite analyses remain to be done. As for 3), it is effectively limited to a type 1 analysis; hence, its useful role has been and is as a standard of comparison with which to judge the convergence of the other approaches. Contrary to statements which have appeared from time to time in the literature, however, there is no known case where the Galerkin or finite difference method has failed to converge although for certain geometries the convergence may be distressingly slow. See Dugundji[11] for a lucid discussion of this approach. One may also envision yet other methods which would appear able to handle the linear problem 1 and 2 more efficiently, such as using the transform calculus for the complete flutter problem; however, none of these has as yet proven workable for 2 and are clearly not applicable to the nonlinear problem, 3 and 4. Hence, it is probable that Galerkin's method and the finite difference approach or, alternatively, the finite element approach will continue to be the most popular methods of analysis and deservedly so. For panels of reasonably simple geometry and construction Galerkin's method is probably the better choice though the versatility and simplicity of the finite difference and finite element approaches may become increasingly attractive for more complicated structures as larger and faster computing machines become available.

One disadvantage of all the foregoing methods is the large number of degrees of freedom needed to accurately describe the behavior of plates of large length/width ratio, a/b. For $a/b > 1$ the flutter mode becomes dominated by increasingly higher modes as $a/b \to \infty$. Thus more and more modes or finite difference elements are required to give an accurate result. Hence, for any given computing device there is an a/b beyond which it is impractical to obtain results. This suggests trying another approach which takes advantage of the special character of the flutter mode for large a/b. A linear model Ref. 22 has been proposed which considers the plate infinitely long, based on the observation that for a high mode flutter the wavelength will be short compared to the plate length. Comparisons of results obtained from such a model with those from analyses which consider a finite length plate have not been encouraging however, especially for high Mach number flows. Quite recently an analysis has been made which explains why the theory of Ref. 22 does not work at high supersonic speeds. See Sec. 4.

2.4 Results of Studies of Particular Plate and Shell Geometries

2.4.1 Flat plate qualitative results

From basic theoretical considerations, e.g., dimensional analysis, the dependence of the plate deflection may be expressed as

$$w/h = w/h(\xi,\eta,\tau;\lambda^*,\mu,M,a/b) \tag{1}$$

In Eq. (1), we state that the ratio of plate deflection to plate thickness w/h is a function of two spatial variables ξ, η, and time τ. It also depends on the parameters, λ^*-dynamic pressure, μ-air/plate mass ratio, M–Mach number and a/b–plate length/width ratio. All quantities are nondimensionalized, (see Nomenclature). If one treats the problem in the linear theory one usually determines the stability or flutter boundary which gives a relation between the parameters which is the dividing curve between stable and unstable motion

$$\lambda^*_{\text{flutter}} = \lambda^*_{\text{flutter}}\,(\mu,M,a/b) \tag{2}$$

Also, one determines the frequency at flutter

$$K_f = K_f(\mu,M,a/b) \tag{3}$$

In the nonlinear problem our primary concern is with peak amplitude and frequency of the nonlinear oscillation hence the results are presented as

$$(w/h)_{\text{peak}} = (w/h)_{\text{peak}}\ (\lambda^*,\mu,M,a/b) \tag{4}$$

$$K = K(\lambda^*,\mu,M,a/b) \tag{5}$$

If in the preceding, (w/h) peak % 0, we obtain the flutter boundary and corresponding flutter frequency.

Theoretical results and, to a lesser extent, experiment indicate that the four basic parameters play primarily the following roles: a) M and a/b together determine the character of the oscillation, i.e., modal content, frequency, etc.; b) the magnitude of λ^*, for fixed M and a/b, determines whether the plate flutters or not and, if it does flutter, the severity of the oscillation; c) the results are generally only weakly dependent on μ for the values of μ normally encountred in aeronautical practice. If the frequency is very high (for example, for large a/b) there is some theoretical evidence to suggest that μ becomes important unless one uses the parameter, $U^* \equiv (\lambda^*/\mu)^{1/2}$, a nondimensional velocity, in place of λ^*. This is unconfirmed experimentally at present, however. It is likely that μ is more important whenever high frequency oscillations are present since μ is to some degree a measure of the importance of the unsteadiness or memory of the flow. Also, in the low supersonic-transonic regime μ may be important for so-called single-degree-of-freedom flutter.

In Fig. 3 is shown a division of the M, a/b plane into various regions where different types of flutter predominate. For high supersonic Mach number and moderate to small a/b, the flutter oscillation is a coupled mode flutter possessing equal amounts of the first and second natural modes and with a frequency between the natural frequencies of these modes, but somewhat nearer the second. At most subsonic Mach number the plate diverges or buckles (flutter frequency is zero) with the first mode dominant. In the low supersonic, transonic range theory predicts a weak first mode* flutter which generally is observed in experiments. This instability because of its weakness is sensitive to structural damping, g, and also μ unlike the stronger coupled mode flutter usually found. Sufficient structural damping will usually suppress this instability entirely leaving only a stronger instability with first and some second mode content. This type of instability is also sensitive to viscous boundary layer effects.

For very large a/b, theory and limited experimental studies indicate the flutter motion is of the traveling wave type with wavelengths on the order of the plate width. Here we use traveling wave in a generalized sense, indicating strong temporal phase shifts between modes.

Available evidence indicates that the transitions between regimes is generally smooth though sometimes rapid, e.g., as one passes through $M = 1$.

Quantitative Results: Now let us consider quantitative descriptions of $(w/h)_{\text{peak}}$ and K as functions of λ^*,μ,M, a/b. In Fig. 4 we present a plot of $(w/h)_{\text{peak}}$ for various M with $a/b = 1.3$ and $\mu = 0.1$ (a representative value).[19] Below

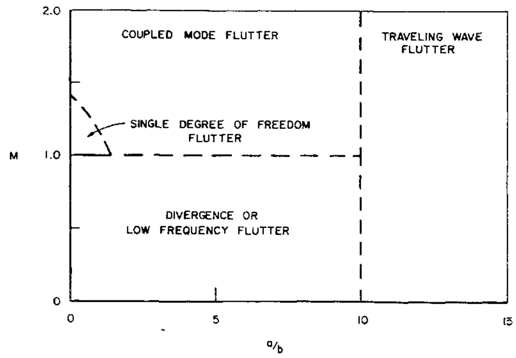

Fig. 3 Regimes of flutter.

* Perhaps a higher mode for larger a/b, see Cunningham.[12]

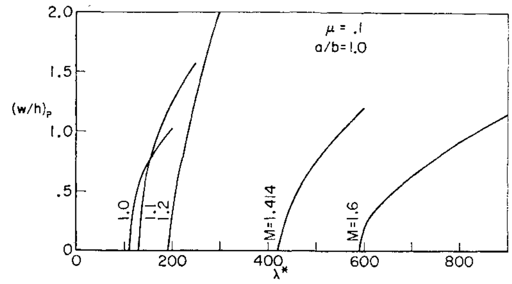

Fig. 4 Limit cycle amplitude vs dynamic pressure.

a certain critical value of λ^* (flutter dynamic pressure, λ_f^*) any disturbance to the plate decays and $(w/h)_{\text{peak}} = 0$ For $\lambda^* > \lambda^*_f$ a periodic (almost sinusoidal) limit cycle oscillation exists with increasing amplitude as λ^* increases. Results are also available for subsonic Mach number and other a/b, but they are omitted here for the sake of brevity. The curves steepen as a/b increases and M decreases until $M \approx 1$ when the trend is reversed. As one penetrates further into the flutter regime (λ^* increasing) the frequency of the oscillation increases, sizable temporal phase shifts between modes occur and, for $a/b = 2$ (not shown), the oscillation takes on an irregular character for λ^* sufficiently large.

As $M \rightarrow 1.0$ for $a/b = 0$, 0.5, 1.0 the flutter mode is dominated by the first natural mode of the plate; however, for $a/b = 2.0$ the first two modes are present in roughly equal amounts. Typically there is a strong (90°) temporal shift between the first and second modes. For higher M the flutter mode is predominately the first two modes in equal amounts with little phase shift.

The effect of μ and g is generally found to be small except for those cases where flutter is a one mode or single-degree-of-freedom phenomenon. For these cases, the flutter oscillation is weak, as evidenced by very slow growth rates of the plate motion in reaching the limit cycle oscillation. Hence μ and g, whose precise values are normally unimportant, become significant. μ also tends to be more important as the frequency of the oscillation increases (flow is more unsteady) and, hence, as a/b increases and also as one penetrates the flutter regime more deeply.

For high Mach number the piston or quasi-steady flow theory is satisfactory (perhaps with a correction for hypersonic effects). This theory predicts that the results depend upon λ^*,M through the single parameter, $\lambda \equiv \lambda^*/M$, for $M \gg 1$. The larger a/b, the larger M must be for this to hold; roughly as $\beta > a/b$. This requirement is because piston theory neglects the three-dimensionality (as well as the memory) of the flow. A more exact inequality could be given on the basis of wavelength of the flutter mode rather than plate length; but, of course, this is not known a priori. In any event, a piston theory flow analysis gives the correct asymptote for M large and in Fig. 5 results are presented from studies for several a/b (Ref. 13). The reader may verify that for $M > 1.5$ these results agree reasonably well with those presented in Fig. 4 if one employs λ^*/β as suggested

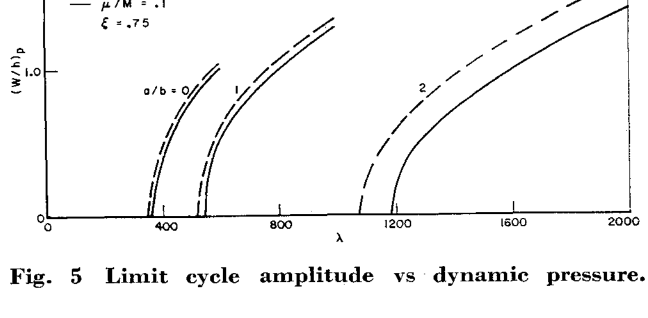

Fig. 5 Limit cycle amplitude vs dynamic pressure.

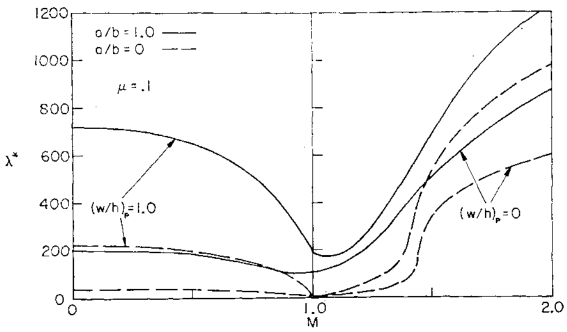

Fig. 6 Dynamic pressure vs Mach number.

by quasi-steady theory rather than λ^*/M, the piston theory result.

The aforementioned results can be viewed in an informative way by a cross-plot which gives λ^* vs M for various $(w/h)_p$ = const. In Fig. 6 such a cross-plot for $(w/h)_p = 0$, 1.0 gives the dynamic pressure, λ^*, at which flutter begins, $(w/h)_p = 0$, and at which the limit cycle amplitude reaches one plate thickness, $(w/h)_p = 1.0$. Note the most critical regime is in the vicinity of $M = 1.0$. For $a/b = 0$, the theory predicts $\lambda^* = 0$. This is due to a combination of factors. The flutter frequency approaches zero, hence the flow is steady; the plate is two-dimensional; the aerodynamic theory is linear and inviscid. All of these factors together give rise to an infinite aerodynamic force. The absence of any one of these factors, in particular if the plate and flow are three-dimensional ($a/b \neq 0$) as for $a/b = 1.0$, 2.0, leads to a finite value of λ^* at $M = 1.0$. This breakdown in the full linearized theory for $a/b \to 0$, $M \to 1.0$ may be remedied by including nonlinear aerodynamic and/or viscous effects. For a/b bounded away from zero (see Ref. 19 for a precise estimate) the linearized, inviscid theory should be satisfactory for sufficiently thin boundary layers.

Note that results for $M < 1.0$ are also presented in Fig. 6. These results are essentially of a divergency character (zero frequency flutter) for $a/b = 0$, 1.0, and a rather low, but finite, frequency flutter for $a/b = 2.0$. In Fig. 7 the flutter frequency at the onset of flutter is given for all M.

It should be mentioned that the results for $(w/h) = 0$ are in generally good agreement with those from linear theory.[12,23]

2.4.2 *Flat plate under transverse and in-plane loads*

In practice, plates will be under external loads of various sorts. Indeed, a principle difficulty in experiments is the control of transverse loads due to a static pressure differential across the plate and in-plane loads due to thermal stresses arising from a temperature differential between plate and boundary support. From the point of view of theory the precise source of the transverse or in-plane loads is unimportant. However, to keep the discussion physically oriented we shall speak primarily of pressure differential rather than transverse load and thermal stresses rather than inplane load. These are also the loads whose effects have been most well documented.

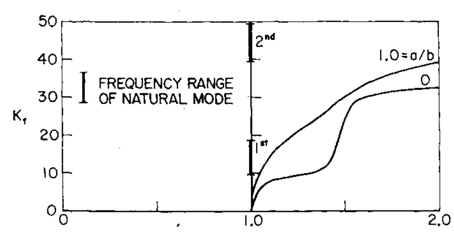

FIG.7 FLUTTER FREQUENCY VS MACH NUMBER

Fig. 7 Flutter frequency vs Mach number.

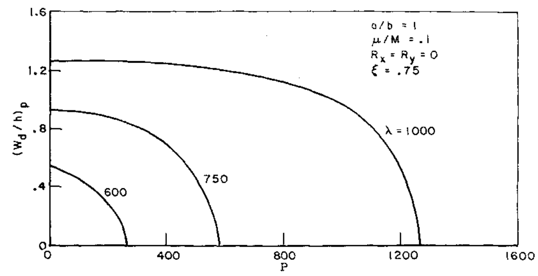

Fig. 8 Dynamic limit cycle amplitude vs static pressure differential.

Flat plate under a static pressure differential: Although it is now generally appreciated that this effect is important in experiments, only relatively few investigations have been made theoretically.[13,16,24] This is because a nonlinear plate theory is required even to establish the flutter boundary. Theory suggests that the effect of a pressure differential may be described by a single new nondimensional parameter

$$P \equiv \Delta p a^4/Dh = (\Delta p/E)(a/h)^4\, 12(1 - \nu^2)$$

In the results to be presented only constant P will be considered; however, a variation of P over the plate surface or with time would create no real difficulty, provided the variation were accurately known.

In the absence of a fluid flow, the plate will deform to some static equilibrium position under the constant pressure differential. With a fluid flow, and for sufficiently large λ^*, this equilibrium position becomes unstable and the plate goes into a (nonlinear) flutter oscillation about the unstable static equilibrium form. Typical results are shown in Fig. 8 for the amplitude of the dynamic or oscillatory component of the flutter oscillation as a function of P for several $\lambda \equiv \lambda^*/M$ when $M \gg 1$ and $a/b = 1.0$. The flutter boundary is determined from Fig. 8 by setting $(w_{d/h})_p = 0$. See Fig. 9. Note the results are symmetric in P; changing the sign of P merely changes the sign of w/h. As may be seen the effect of $P \neq 0$ is to raise λ^*_f. Physically this is because the plate is stiffened by the tension induced due to the plate stretching as it bends under the pressure differential load. Results have also been obtained for $M \to 1$. The interesting result here is that the minimum λ^* and the Mach number at which it occurs depends upon P. Ventres et al.[25] have also shown the importance of in-plane edge restraint and the more complicated behavior for larger a/b.

Flat plate under thermal stress: Depending on whether the plate temperature is higher or lower than that of its boundary support, compressive or tensile stresses will be introduced into the plate. Tensile stresses or compressive stresses below the buckling load of the plate, while changing the effective plate stiffness and hence its flutter behavior, do not create any essentially new phenomena. The most interesting and important consequence of thermal stresses arises when they are sufficiently large in compression to cause

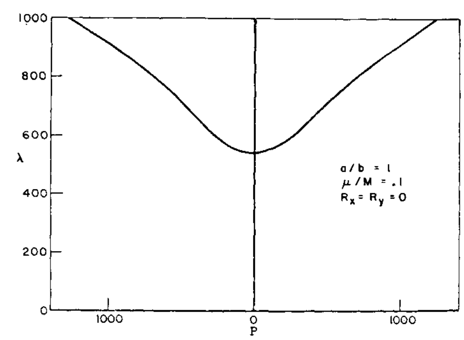

Fig. 9 Flutter dynamic pressure vs static pressure differential.

plate buckling. The flutter of a buckled plate was considered in a partial nonlinear analysis several years ago by Fung[14] and Houbolt[26] and more completely and more recently by Kobayashi,[17] Fralich,[16] Dowell,[13] and Ventres and Dowell.[25] There is also a sizable Russian literature.[14] The results presented here are from Ref. 13.

For a plate under general applied in-plane loads the following dimensional parameters must be added

$$N_x^{(a)},\ N_y^{(a)},\ N_{xy}^{(a)}$$

For a constant temperature difference between plate and boundary support, ΔT,

$$N_x^{(a)} = N_y^{(a)} = -Eh\alpha\Delta T/(1 - \nu)$$

$$N_{xy}^{(a)} = 0$$

The corresponding dimensionless parameters are

$$R_x \equiv N_x^{(a)}a^2/D,\ R_y \equiv N_y^{(a)}a^2/D, \text{ or}$$

$$T \equiv 12\ (1 + \nu)\ (a/h)^2\alpha\Delta T$$

The variable temperature differential case[27] has also been treated using linear plate theory and could be extended to the nonlinear analysis as well.

Typical results are shown in Fig. 10 for limit cycle amplitude vs λ when $M \gg 1$ and $a/b = 1.0$. The case $R_x = R_y = -T$ is considered. For a given R_x there are two branches to each curve when $R_x < -2\pi^2$. $R_x = -2\pi^2$ is the classical Euler buckling load for a square plate. The branch for smaller λ^*/M is a buckling branch that is the classical (nonlinear) buckling problem as modified by the fluid forces. The frequency of the limit cycle oscillation is zero. Note, however, from elementary physical reasoning that if w is a solution then, so is $-w$ for $P = 0$. Thus an oil-canning effect is possible if the panel is continually disturbed and shifts from one nonlinear static buckling configuration to the other. As may be seen, the flow increases the buckling load. The branch for higher λ^*/M is a flutter branch as modified by the applied in-plane load, consisting of a limit cycle with a nonzero frequency. Clearly the flutter type instability is aggravated by a compressive in-plane load as far as peak deflections (and also stresses) are concerned. If the in-plane compressive load is below the Euler value, the effect on the flutter branch is only moderate.

The flutter and buckling boundaries for the plate may be determined by setting $(w/h)_p = 0$. They are given in Fig. 11. Below the nearly horizontal line emanating from the intersection of the flutter and buckling boundaries ($\lambda \approx 200$, $R_p \approx -4\pi^2$), the panel is buckled in a nonlinear equilibrium configuration that is itself dynamically stable. Above the line the plate undergoes a limit cycle oscillation (nonlinear flutter). It will be noted that there are basically two types of flutter, an essentially simple harmonic oscillation and a periodic but nonsimple harmonic oscillation. This latter type of oscillation is indicated by dotted lines in Fig. 10. Also it was determined that for sufficiently large compressive loads the motion, while approximately simple harmonic, may

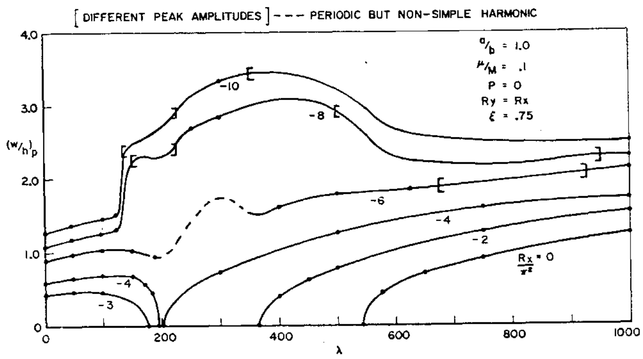

Fig. 10 Limit cycle amplitude vs dynamic pressure.

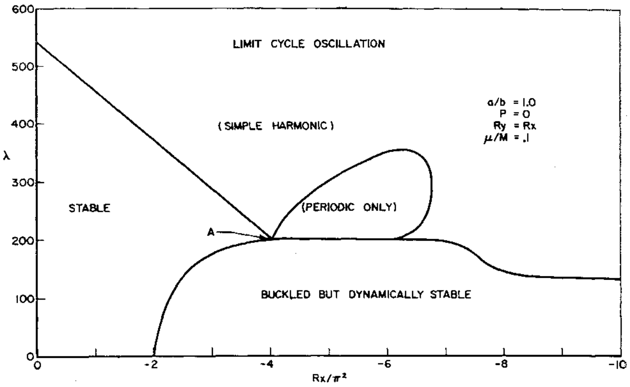

Fig. 11 Stability regions.

have different negative and positive peak amplitudes (see Fig. 10). See Ref. 13 for a more thorough discussion of the various types of plate motion. Note the present results for $R_x/\pi^2 = -8$ differ from those of Ref. 13, the latter contained a numerical error.

Because of the shape of the flutter boundary in Fig. 11, which is also characteristic of much of the experimental data, it has been suggested that linear theory may be used to determine the most critical (lowest) λ which occurs at the intersection of the flutter and buckling branches. However, the full nonlinear theory is required to assure that the minimum λ^*/M for flutter has been found and, of course, the nonlinear theory is essential to investigate the limit cycle region above the flutter boundary. In fact, from Fig. 11, it is seen that for $R_x/\pi_2 = -8$, λ_f is lower than for $R_x/\pi^2 = -4$. Results have been computed using the full aerodynamic theory for $M < 2$, also, i.e., a type 4 analysis.

Ventres et al.,[25] have recently shown a number of interesting results for loaded plates. We shall defer a detailed discussion of their results until later when comparisons of their theoretical results are made with experiment.

However, among their important findings are: 1) the quantitative and qualitative importance of the degree of in-plane edge restraint for pressurized or buckled plates; 2) the possibility of lowering the flutter boundary if a buckled plate is subjected to a large disturbance. It is clear from such results that the flutter of loaded plates is a basically more complex problem than that of unloaded ones.

2.4.3 Curved plates

It will be convenient to divide the discussion into two parts, one for plates with cross-stream curvature and the other for streamwise curvature. The most general case of curvature in both directions (and possibly twist) appears to have received little attention until recently. The following dimensional parameters, $\mathcal{R}_x$, $\mathcal{R}_y$, $\mathcal{R}_{xy}$, x and y curvature plus twist lead to the nondimensional parameters

$$\Gamma_x \equiv a^2/\mathcal{R}_x h \approx 8\ H/h,\ \Gamma_y \equiv a^2/\mathcal{R}_y h,\ \Gamma_{xy} \equiv a^2/\mathcal{R}_{xy} h$$

We shall restrict the discussion to constant curvature since only for this case are substantial results available.

Cross-stream curvature: For curvature in the direction perpendicular to the flow the problem, within the theoretical framework of shallow shell theory, is essentially the same as the flutter of a portion of a circular cylinder. Noting this Voss[28] inter alia, carried out a theoretical investigation (type 1 analysis) which demonstrated a large stiffening and hence stabilizing effect due to curvature.

Subsequent to Voss' work little has been done although Bolotin[15] has formulated a type 3 analysis for a plate of general curvature. Quite recently Dowell[29] has carried through a type 3 analysis to the point of obtaining numerical results. Perhaps the most significant result of this work is the demonstration that in-plane edge restraint is an important parameter

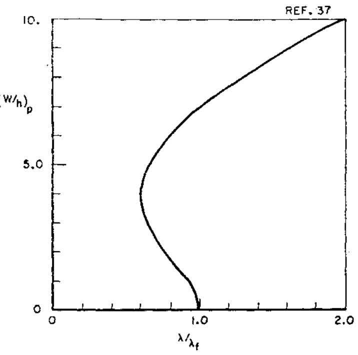

Fig. 12 Limit cycle amplitude vs dynamic pressure.

for such plates. In particular Voss' conclusion is only true for plates with free or nearly free in-plane edge restraint. For completely restrained edges, curvature decreases the flutter dynamic pressure even though it stiffens all plate natural modes. The stiffening, it turns out, is selective and the fundamental mode frequency is raised to a greater degree than the second. Hence, the frequency (squared) difference is decreased and flutter is more likely.

Experiments at NASA Langley[30,31] have confirmed the stabilizing effect qualitatively and brought out the additional complication due to static pressure loading on a curved panel. A pressure loading which tends to flatten the plate may lead to snap through buckling. If this occurs, the problem is considerably complicated. The proceeding also implies, of course, that a zero static pressure loading will not, in general, be the most critical flutter condition for a curved plate as it was for the flat panel. The experimental indication of stiffening due to curvature implies the in-plane support was very flexible; this agrees with other results obtained for flat plates of typical construction.

Streamwise curvature: Yates and Zeydel[32] have carried out a type 1 analysis. Fung[14] had previously considered the problem in the context of the effect of imperfections on the flutter of buckled plates. In Ref. 32, it was concluded that the effect of small curvature was destabilizing.

Recently, a type 3 analysis for the problem has given a number of interesting results.[7,29] The necessity for considering preflutter deformation under static aerodynamic has been shown theoretically. It was also shown that the flutter deflection amplitudes were on the order of the maximum rise height of the curved plate. Also see Ref. 29 for results with both streamwise and cross-stream curvature.

2.4.4 Cylindrical shell

While not studied to the same extent as the flat plate, some significant work has been done on the cyclindrical shell. Chronologically, the first (linear) theoretical investigations were of infinitely long cyclindrical shells.[33,34] Recently, these have been extended and improved.[35] Unfortunately, as for the flat plate, there is at present no assurance that such a model is useful even for cylinders of large length to radius. Subsequently, both linear and nonlinear analyses have been carried out for finite length shells using "piston theory" for the fluid forces. Of the type 1 analyses, that of Voss[28] is most complete. There are, of course, various shell theories commonly used. Voss has used that due to Goldenvieser including the axial and circumferential as well as radial inertial terms. The in-plane inertias were shown to be of importance for axisymmetric motion. A major difficulty of the cylindrical shell (and also curved plate) is that the lowest circumferential (or cross-stream) mode may not be the most critical, as it is generally true for the flat plate. Voss has shown that either the axisymmetric or a high asymmetric circumferential mode may be the most critical with regard to flutter.

Olson and Fung[34] have carried out a nonlinear analysis, type 3, using Donnell's shell equations (essentially the shallow shell approximation) which are generally satisfactory for the higher, though not the lower, circumferential modes. Their analysis uses only two modes and, hence, is of a qualitative character. However, they find the interesting result that for high asymmetric mode flutter, the flutter mode may jump from one circumferential mode to another as one penetrates further into the flutter regime.

Subsequently, Evensen and Olson[35] have improved upon the nonlinear analysis of Olson and Fung by including additional modes and higher order nonlinear terms. They show that circumferentially traveling wave flutter is possible, in qualitative agreement with experiment. They also indicate that flutter is possible at dynamic pressures significantly below the linear flutter boundary. In Fig. 12, a representative result is shown for flutter amplitude vs λ.

Recently, Carter and Stearman[36] have also presented a partial nonlinear analysis using the (nonlinear) Donnell equations and a modified form of piston theory. In this work, the condition of the cylindrical shell under a static pressure differential is determined using the nonlinear theory after which a linear perturbation flutter analysis is performed about the loaded shell condition to determine the stability boundary. This work is further refined by Barr and Stearman.[37]

No type 2 or 4 analyses have been carried out.

2.4.5 Cavity effect

Before turning to comparisons between theory and experiment, a brief discussion will be given of the cavity effect. Physically, a shallow closed cavity beneath the plate leads to a stiffening of the plate due to the compressibility of the cavity air. (There is also a virtual mass effect which is negligible under most conditions.) The stiffening is selective in that it effects primarily the fundamental cavity mode by raising its effective natural frequency. This tends to raise λ^*_f if one has single degree of freedom flutter in the first mode and lower λ^*_f if flutter occurs as a result of coalescence or frequency merging of the first two plate modes. A single new parameter is introduced (if the cavity air is as the same as freestream conditions), cavity depth/plate length, d/a. The good agreement between theory and experiment for natural plate vibrations with a systematic cavity variation indicates this aspect of the problem is well understood.[5]

2.4.6 Structural damping

The subject of structural damping has provided a topic for much discussion. This is largely due to the discovery that some types of damping could theoretically provide a destabilizing effect on flutter. Bolotin[15] and Dugundji,[11] among others, have discussed the difference between damping proportional to velocity and damping incorporated through a complex modulus of elasticity, e.g., the $[1 + ig]$ type, the latter corresponding to a viscoelastic material. The essential

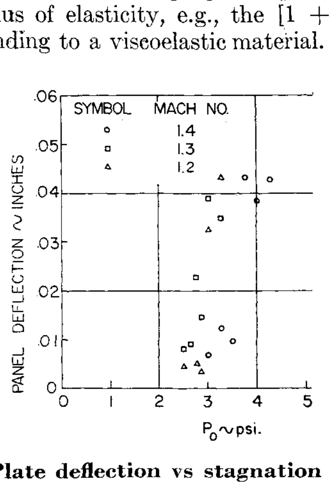

Fig. 13 Plate deflection vs stagnation pressure.

difference between the two models of damping is the manner in which the damping varies from (natural) mode to mode. It turns out (as one would expect) that the velocity damping increases the flutter dynamic pressure; what is perhaps unexpected is that the complex modulus damping can under some circumstances lower the flutter dynamic pressure. For typical magnitudes of damping coefficients, say $g = 0.01$, the flutter dynamic pressure may be reduced by one or two percent for the coupled mode type of flutter. This paradoxical result has led to many attempts at explanation. Without going into the theology of this matter, we would like to make two points here:

1) Where theory indicates that damping may be destabilizing (coupled mode flutter) the quantitative effect is very small, see the aforementioned; on the other hand, when the quantitative effect is significant (single-degree-of-freedom flutter) the two types of damping give substantially the same result, i.e., the flutter dynamic pressure is increased and by the same amount. In the present paper we use the complex modulus form of damping.

2) Measurements of structural damping on panels of typical construction indicate that damping is most accurately described by a velocity damping whose magnitude is inversely proportional to frequency. If the effect of structural damping is thought to be important, only measurement of damping offers a rational approach.

3) Some structures under certain loading conditions may be abnormally sensitive to variations in structural damping and/or aerodynamic damping. If theory or experiment indicates that the flutter condition is sensitive to damping, it may be taken as prima facie evidence of a general sensitivity of the configuration. If at all possible the configuration or loading should be modified to eliminate this sensitivity.

3. Experimental Results and Correlation with Theory

3.1 Flat Plate

No experimental data have been obtained with the express purpose of investigating the post-flutter (nonlinear) regime. However, in determining the flutter boundary some limited data have been taken in this regime. In Fig. 13 is shown a plot of maximum deflection amplitude (peak to peak) vs stagnation pressure (dynamic pressure) for a flat plate.[5]

Note the plate thickness was 0.025 in. Below the flutter boundary the plate amplitude is small compared to the plate thickness. Above it the plate amplitude is on the order of the plate thickness. This is in qualitative agreement with theory but additional data to larger dynamic pressure are needed for an adequate quantitative comparison.

With data of the type shown in Fig. 13 one may determine (within a certain band of q) the flutter boundary. All of the available quantitative comparisons between theory and experiment are for the boundary. In Fig. 14 are shown such comparisons for plates of three different thicknesses. In

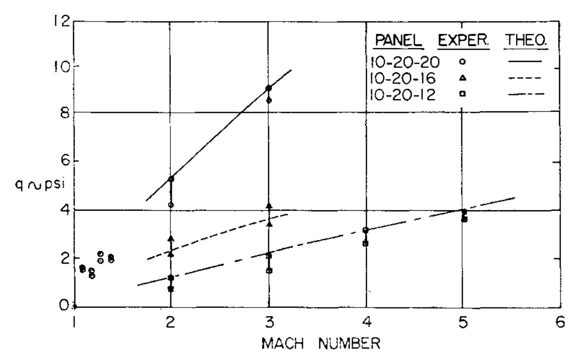

Fig. 14 Flutter dynamic pressure vs Mach number.

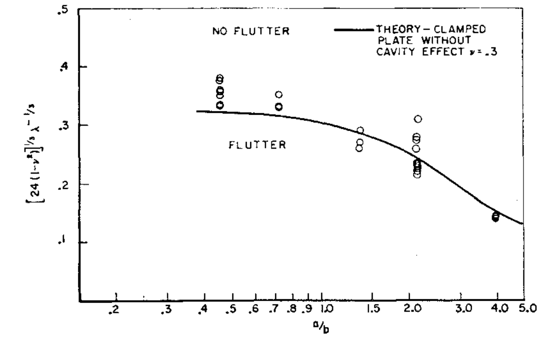

Fig. 15 Thickness ratio vs length/width ratio.

dimensional form, we have q_f vs M. As may be seen the agreement is quite satisfactory. These data are for $a/b = 0.46$; similar data with similar agreement are presented in Ref. 5 for $a/b = 0.73, 1.37, 2.18$, and 4.11. The plates are aluminum and clamped on all edges. The theory is a type 1 analysis with cavity effect and measured natural frequencies. Most of these data, as in Fig. 14, are for $M \geqslant 2$. For these high Mach numbers, one may use the correlation parameter λ as previously discussed, and in Fig. 15 are presented all the data of Ref. 5 for $M \geqslant 2$ in nondimensional form as compared with theory for an ideal clamped plate, neglecting the cavity. The parameter actually used $\lambda^{-1/3}$ is essentially the thickness/length ratio required to prevent flutter (above the boundary) vs length/width ratio. This parameter, often used in the literature, is more commonly employed for design situations. Each point represents a different plate and Mach number. As may be seen agreement is generally satisfactory, particularly in view of the large number of dimensional variables contained in λ. The scatter shown is due to a number of factors but primarily is a reflection of individual construction differences in the models and neglect in the theory of the cavity effect. Construction differences could result in a $+10\%$ scatter and the cavity in $+5 \rightarrow 10\%$. Additional data are available which also are in reasonable agreement with theory; see Dixon[23] for an interesting and useful discussion.

A smaller amount of data have been obtained in Ref. 5 for $M < 2$ which show poorer agreement with theory. Recently, Muhlstein, Gaspers, and Riddle[41] have conducted experiments to determine flutter boundaries of thin, flat plates at low supersonic Mach number, $M = 1.1 - 1.4$. A careful, systematic examination of boundary-layer effects on panel flutter has been made. Extrapolating their data to zero boundary-layer thickness Dowell[42] has compared their experimental results with theoretical results from Ref. 5.

In Fig. 16 the experimental data of Ref. 41 and the theory of Ref. 5 for a length-to-width ratio of one-half, $a/b \doteq 0.5$, are presented in terms of flutter dynamic pressure λ_f^* vs Mach number M. In the legend the three numbers designating each plate are length in inches, width in inches and thickness in thousandths of an inch. For magnesium it is assumed that

$$E = 0.56 \times 10^7 \text{ psi}, \nu = 0.35$$

Experimental data are given for zero boundary-layer thickness (as extrapolated) and the maximum boundary-layer thickness of the tests. Two theoretical curves are given from Ref. 5. One curve is for zero structural damping, and the other for a maximum possible structural damping, $g = 0.05$. The values of mass ratio used for the theory are those measured in Ref. 39 for zero boundary-layer thickness, i.e., they are extrapolated values. The theory is not particularly sensitive to μ relative to its sensitivity to λ^*. On the other hand, the value assumed for structural damping can be important, although $g = 0.05$ is an upper bound on the possible amount of structural damping. $g = 0.01$ would be more representative for the present plates. The plate edges are clamped.

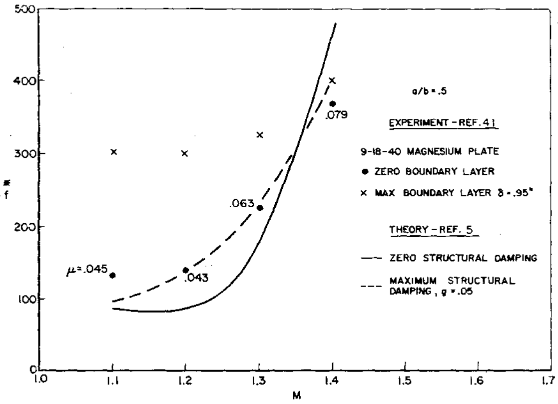

Fig. 16 Flutter dynamic pressure vs Mach number.

As can be seen, theory and experiment (for zero boundary-layer thickness) are in generally good agreement. Inclusion of structural damping in the theory gives a modest improvement in the agreement. Conversely, for nonzero boundary-layer thickness, the experimental data are considerably different from theory particularly at $M \leq 1.3$. Hence, one concludes that much of the difference previously observed between theory and experiment can be attributed to boundary-layer effects.

The poorer agreement between theory and experiment noted in Ref. 5 is thus now explained. Also see the earlier discussion by Lock and Fung[8] who first discussed this possibility. Finally, it should be stated the agreement on flutter frequency is generally as good as or better than that on flutter dynamic pressure.

For larger a/b limited available theoretical, type 1 and 2 analyses, and experimental data for $M = 3$, $a/b = 10$ show reasonable agreement. There are indications from theory that for large a/b, q_f becomes independent of a and a new parameter $\lambda^*_b = 2qb^3/D$ with a replaced by b would be appropriate. This also seems physically plausible. Generally speaking, plates of large a/b are relatively unexplored. Again see Ref. 23.

For a different and somewhat complementary discussion of theoretical-experimental correlation at supersonic speeds of flat plates the report of Dixon[23] is recommended to the reader. As the careful reader will note there are some minor points of disagreement berween the present discussion and Ref. 23. There is general agreement, however.

From the above, one may conclude that the principle needs are for 1) additional experimental data for large a/b and 2) data extending well into the nonlinear regime. Data for large a/b are particularly scarce even for the flutter boundary. Generally speaking, available theoretical tools would appear satisfactory with the exceptions of 1) boundary-layer effects and 2) large a/b plates. See Sec. 4 with regard to the latter two aspects.

3.2 Flat Plate Under Transverse and In-Plane Loads

The following discussion is taken largely from Ref. 25.

3.2.1 Natural frequencies of pressure loaded plates

A plate exposed to a transverse pressure load undergoes a static deformation that causes stretching in the middle surface of the plate. The membrane stresses associated with this stretching stiffen the plate, raising its natural frequencies of vibration. In-plane boundary support flexibility will relieve the stresses and so change the natural frequencies of the loaded plate. Since it is known from the linear theory that the stability of a plate in a uniform airflow is governed by the relative spacing of its natural frequencies, it is important that the in-plane boundary conditions be known, and that their effect on the frequency spectra be understood. Moreover, the accuracy with which the natural frequencies can be computed provides an excellent test of the ability of the theory to properly represent the elastic behavior of the plate.

Figure 17 shows the effect of a static pressure differential on the lowest three natural frequencies of a panel built and tested by Ventres and Dowell.[25] The panel consisted of an aluminum plate, $20 \times 10 \times 0.05$ in., bonded to a rectangular frame built up of aluminum rails welded together at the corners. Also shown in Fig. 17 are the natural frequencies computed from a Galerkin analysis for the limiting cases of zero and complete in-plane restraint.

The modal equations were numerically integrated to find the static reflection under the pressure loading, and then linearized about this equilibrium configuration to determine the natural frequencies in the usual way.

Note that theory and experiment agree quite well at $\Delta p = 0$, but that the assumption of complete in-plane restraint considerably overestimates the frequencies for $\Delta p \neq 0$. The experimental data lie closest, in fact, to the curves for zero in-plane restraint. Presumably then, the rails to which the plates edges were bonded were sufficiently rigid to restrain them against rotation (thereby maintaining a clamped boundary condition), yet were unable to provide complete restraint against in-plane motions at the edges. Static theory and pressure loading tests further confirmed this.

3.2.2 Flutter of pressure loaded plates

In this section the influence of in-plane edge restraint on the flutter behavior of loaded plates will be investigated by determining how a pressure differential alters the stability boundaries of plates with and without in-plane edge restraint. Comparisons will also be made with experimental results for flutter boundaries of pressurized plates reported in Ref. 5.

The two panels of interest here measured 8.5 in. by 18.5 in. between supports, and were 0.02 and 0.025 in. thick. The thinner of the two (designated as panel 10-20-20) was tested with its short side parallel to the flow, so that $a/b = 0.46$. The other (panel 20-10-25) was tested with its long side parallel to the flow, so that $a/b = 2.18$. As mounted in the tunnel, a cavity 2.5 in. deep existed beneath the panels. This cavity was pressurized during the flutter testing. Stability boundaries were determined as a function of the pressure differential between the cavity and a static pressure tap in front of the panels. The measured flutter boundaries were not symmetric about $\Delta p = 0$, presumably due to pressure gradients over the plates or to flow misalignment. Therefore, the data presented (in dimensionless form) in Figs. 18 and 19 have been adjusted to make them symmetric about $\Delta p = 0$. This effectively assumes the existence of some equivalent constant Δp.

In calculating theoretical flutter boundaries for comparison with these results, the effect of the cavity has been included.

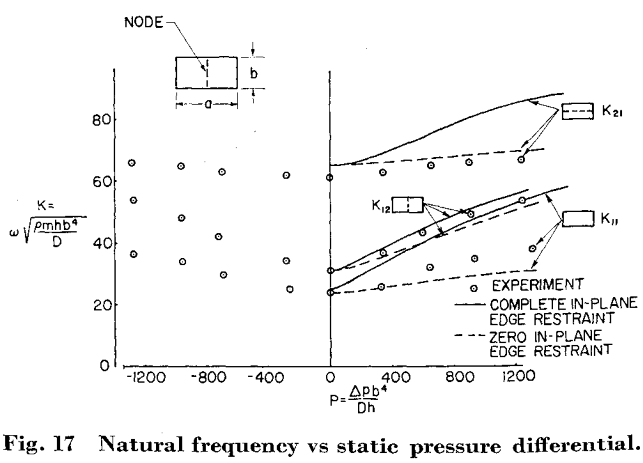

Fig. 17 Natural frequency vs static pressure differential.

In addition, experimentally determined values for the natural frequencies (for $\Delta p = 0$) of the two panels, taken from Ref. 5, were used to account approximately for the effects of imperfect panel construction. These frequencies characterize the linear elastic behavior of the panels. They were inserted into the modal equations to correct the linear stiffness of the plates. The theoretical flutter boundaries calculated for zero and complete edge restraint are shown in Fig. 18 ($a/b = 0.46$) and 19 ($a/b = 2.18$) along with the experimentally derived flutter boundaries.

Note first of all that the relative positions of the stability boundaries for zero and for complete in-plane restraint are reversed in the two figures. Some understanding of these rather surprising results can be obtained by studying the natural frequency spectra of the plates. For $a/b = 2$, flutter is caused by aerodynamic coupling between the first two chordwise natural modes. The stability of the panel is dependent on the separation between these frequencies. As seen in Fig. 17, the dependence of K_{12} on Δp is not sensitive to the in-plane boundary conditions. On the other hand, K_{11} increases rapidly with Δp for the case of complete restraint, but more slowly for zero restraint. The separation between the two frequencies is thereby increased for zero restraint, so that the plate is stabilized, and is somewhat decreased for complete restraint, with the result that the plate is slightly destabilized.

It should be recognized that the behavior of the plate natural frequencies under a pressure load depends not only on the membrane stresses caused by the load, but on the induced middle surface curvature as well. For restrained plates, the effect of the tension is predominant, but for unrestrained plates the influence of the curvature must be considered also. The best agreement between theory and experiment in Figs. 18 and 19 is obtained by assuming zero edge restraint

3.2.3 *Buckled plates*

In-plane compressive loads can cause a plate to buckle, thereby altering its behavior in a uniform airflow. The buckled plate has a curved middle surface and is subjected to mid-plane stresses that depend in a complicated manner on the nature of the in-plane loading, the in-plane boundary conditions, and the external flow. The flutter motion is no longer simple harmonic and may not be perfectly periodic. Moreover, the presence of more than one buckled equilibrium configuration lends some credence to the existence of sustained flutter oscillations below the linear flutter boundary. Such motions have been computed numerically for buckled two-dimensional plates by Fung.[14]

Most experimental work has been done on plates buckled by uniform thermal expansion. Reference 9 is the latest in a series of experiments by NASA Langley. Here comparisons will be made between the a type 3 theory[25] and experimental results from Ref. 9 for plates with $a/b = 2.9$, buckled by

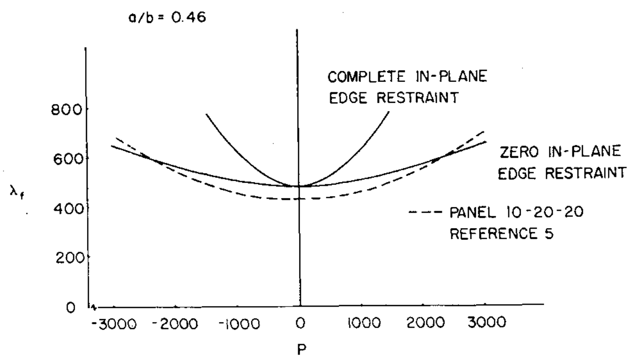

Fig. 18 Flutter dynamic pressure vs static pressure differential.

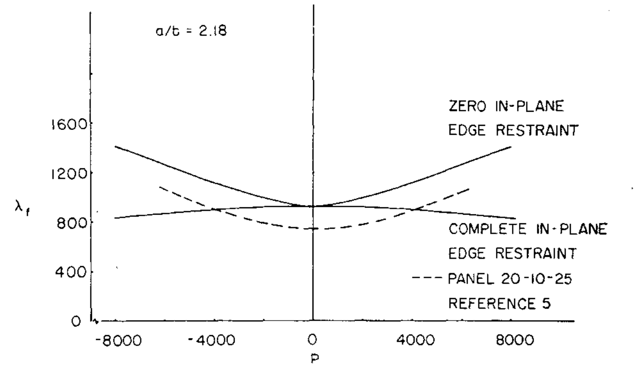

Fig. 19 Flutter dynamic pressure vs static pressure differential.

thermal expansion. The testing was done at a Mach number of 3, and at constant dynamic pressure. The plates were heated by the airflow so that the in-plane loading increased with time, exceeding the buckling value before termination of the run. With but few exceptions flutter occurred prior to buckling, and then ceased at some point subsequent to buckling.

In this sort of test the in-plane boundary conditions affect not only the large-deflection behavior of the plate, but the nature of the in-plane loading as well. If the edges are completely restrained against in-plane motion, the in-plane stresses $N_x^{(a)}$ and $N_y^{(a)}$ are equal, and

$$N_x^{(a)}a^2/D = N_y^{(a)}a^2/D = 12\,(1 + \nu)(a/h)^2\alpha\Delta T$$

where ΔT is the uniform temperature differential between the plate and its support. The parameter

$$R_T \equiv 12(1 + \nu)(a/h)^2\alpha\Delta T$$

will be used here to describe the thermal loading on the plate, even though it may not be fully restrained. If the plate is not fully restrained, then the in-plane stresses due to thermal expansion will generally be unequal. The plates tested in Ref. 9 were riveted to their support frames. Apparently no attempt was made to determine experimentally the effective boundary support flexibility of the resulting structure.

The assumption of complete in-plane edge restraint produces flutter boundaries in very poor quantitative agreement with experiment. It was thus decided to select values of edge restraint so that buckling would occur for the same R_T experimentally and theoretically. This is equivalent to assuming that the airflow does not affect the static Euler buckling load. In consideration of the riveted edge construction, the edge restraints in the two directions were assumed to be equal. Using nonlinear stiffness terms based on such considerations, the stability boundaries shown in Fig. 20 were computed.

In Fig. 20 two flutter boundaries are shown in the buckled regime. The linear or small disturbance flutter boundary was computed using a buckled equilibrium configuration as an initial condition for the numerical integration of the modal equations. On the other hand, calculations proceeding from a flat plate initial condition produced sustained flutterlike oscillations below this linear flutter boundary. Since the buckled shapes involved deflections on the order of several plate thicknesses, this is physically equivalent to applying a large disturbance to the plate. The numerical results obtained suggest that sustained flutter oscillations are possible for λ, R_T combinations above the shaded area in Fig. 20. Note that this large disturbance flutter boundary is in excellent agreement with the experimental data, since most of the data in that region are flutter stop points. The existence of two flutter start points in the same region is not explained by the large disturbance flutter boundary, however.

The flutter motion computed from theory agrees qualitatively with descriptions of the behavior observed experi-

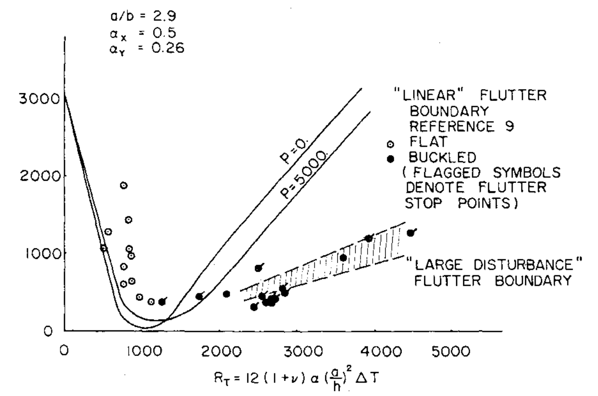

Fig. 20 Flutter dynamic pressure vs temperature differential.

mentally. The plate motion is not simple harmonic, nor strictly periodic, but the peak deflections attained are repetitive. The motion is of the traveling wave variety, with two half wavelengths along the length of the plate. The buckled equilibrium shapes computed below the "large disturbance" flutter boundary consisted of two half wavelengths as well. The frequency of flutter (not shown) is in generally good accord with the experimental data for all values of R_T.

3.3 Curved Plates

The experimental data which are available, due to Anderson,[40] are for square plates clamped on four edges with streamwise curvature. The available theoretical results are for two-dimensional clamped plates (including the static aerodynamic loading).[7] In Fig. 21 the experimental and theoretical flutter boundaries are shown in terms of λ vs H/h ($\Gamma_x \approx 8H/h$). There is qualitative agreement between theory and experiment as can be seen. Interestingly enough the largest discrepancy is for a flat plate, $H/h = 0$. Also shown, is a theoretical result for a square plate when $H/h = 0$, i.e., a flat plate. Note this only gives a modest improvement in the agreement between theory and experiment.

The question arises as to what are the physical sources of the several quantitative discrepancies between theory and experiment. First of all the tested plates[40] were extremely thin, $h = 0.008$ in., and this undoubtedly made them very sensitive to manufacturing imperfections and possibly thermal stresses. Secondly, the in-plane edge supports may have had some flexibility which would effect the curved plates and also the flat plate under pressure loading. Indeed, the weight of the plate itself may have given rise to a significant static loading for such a thin plate. (Lemley[43] has investigated this possibility in some detail.)

Finally (having discussed the quantitative differences) we wish to emphasize two other features of the flutter motion determined theoretically which are in good qualitative agree-

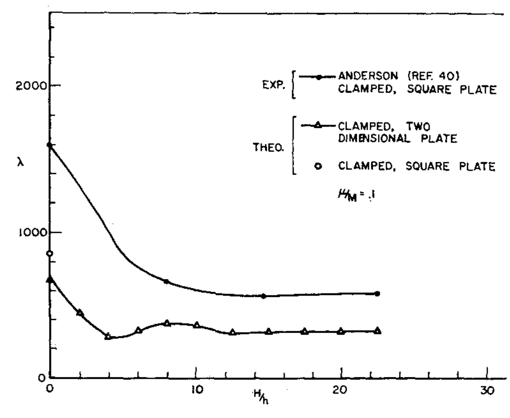

Fig. 21 Flutter dynamic pressure vs plate rise.

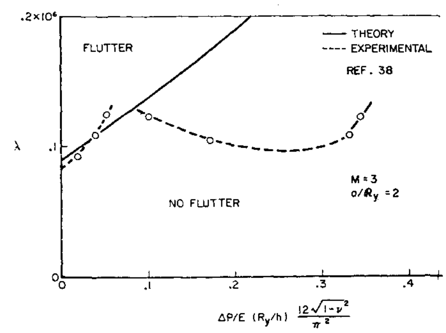

Fig. 22 Flutter dynamic pressure vs static pressure differential.

ment with the experimental flutter data. Firstly, the theoretical flutter amplitude is on the order of, H, the rise height of the plate in good agreement with experiment.[40] Secondly, the plate deformation shapes under static aerodynamic load just prior to flutter are pulled out toward the trailing edge and pressed in toward the leading edge, again agreeing with the experimental evidence.

3.4 Cylindrical Shell

Stearman et al.[44] and later Olson[45] have conducted experiments at high supersonic Mach number, $M = 3$. More recently, Stearman[46] has reported additional tests at low supersonic speeds, M-1.2-1.5. The high Mach number experimental data have been compared to the theoretical results of Refs. 36 and 38. See Fig. 22 taken from Ref. 38 which gives λ vs a static pressure parameter. The latter is essentially $P/\Gamma_y{}^2\pi^2$. There is general agreement on the character of the flutter oscillation, a high asymmetric mode flutter; the agreement on flutter dynamic pressure is also reasonably good for an unpressurized shell but becomes poor as the static pressure differential is increased. Caution is also advised in view of the use of piston theory fluid forces which are much less accurate for cylindrical shell than a flat plate.[47-49] Olson has also used the full aerodynamic forces on an infinitely long cylinder for a finite length shell analysis which leads to much poorer agreement with experiment. For the low supersonic regime, the experimental data indicate the flutter mode is of a low circumferential mode traveling wave type. Hence, Stearman[46] has compared the results with those of the infinitely long shell analysis of Miles[34] which was developed for this type of flutter. Generally, the agreement is not good for flutter dynamic pressure, the theoretical values being much less than the experimental ones. The theory of 35 gives a similar result. The slender body approximation, which has been suggested for high circumferential mode flutter,[47,48] was also used by Stearman to compare with experiment and, as might be expected, is in poor agreement with the experimental data giving values of flutter dynamic pressure considerably higher than the experimental data.

Despite the fact that the fluid forces have been determined using the full linearized potential theory,[47-49] only various asymptotic approximation to these forces have been used even in the linear analyses, as indicated above. One may anticipate, however, that the more complete theory will be used in the near future for both linear and nonlinear analyses. Hopefully, this will help clarify the rather confused picture which presently exists with respect to the relation of theory and experiments.

With regard to improvements in the structural model, Barr and Stearman[39] have shown that cylinders are sensitive to imperfections on the order of the shell thickness. Although the precise nature of the imperfections are unknown, qualitatively speaking they appear to account for some of the differences between theory and experiment.

4. Future Research

4.1 Improvements in the Aerodynamic Theory and Experiment

We shall discuss two possible improvements here with respect to 1) boundary-layer effects and 2) local flow effects.

4.1.1 Boundary-layer effects

From order of magnitude considerations, one may infer that if the boundary-layer thickness is on the order of the flutter wave length, viscous effects will be important. Another parameter which is of importance is the ratio of boundary-layer thickness to plate amplitude. To assess its importance requires a nonlinear theory. Despite considerable work[50–57] in this area, only recently has a realistic flutter analysis been carried out.[58] In order to simplify the flutter calculation while accounting for viscous effects, most investigators have examined infinitely long plates (usually of infinite width as well) or shells and made various rational and/or irrational assumptions concerning the fluid behavior. The essential weakness of most studies is that the analysis does not reduce (in the absence of the boundary layer) to one of known consistency and accuracy. (Actually Refs. 54–56 do not consider panel flutter per se.) Hence, the improvement due to the addition of the boundary layer has been gained at the expense of something else, usually the neglect of finite plate or shell dimensions. In spite of the aforementioned weakness, these studies have been most helpful in pointing the way in which more realistic studies must proceed. In principle, the solutions available for infinite wavy walls only need to be superimposed in an appropriate Fourier manner to obtain the solution for a finite plate. Zeydel[57] has formally generalized the two-layer potential flow model of Anderson and Fung[50] to many layers and finite dimensions using Fourier superposition. No quantitative results were obtained, however. In a very recent analysis, Dowell[58] has developed a solution for a shear layer model over a plate of finite dimensions. Quantitative results have been obtained for boundary-layer effects on the flutter of a two-dimensional plate which are in fair quantitative agreement with the experimental data of Ref. 41 for $a/b = 0.5$. This analysis can be readily generalized to three dimensions and offers considerable encouragement with respect to our ability to handle substantial boundary layer effects of the type shown in Fig. 16. Some additional experimental studies modeled after those of McClure[57] would be most helpful in assessing recent theoretical advances.

4.1.2 Local flow effects

At hypersonic speeds, aerodynamic nonlinearities or pseudo-nonlinearities may become significant. Of the various possibilities, local steady flow conditions over the panel, significantly different from those in the freestream, are apt to be most important for panel flutter. It has generally been believed that the use of the local flow pressure, density and Mach number, in place of the freestream values in the quasi-steady or piston theory, was a satisfactory approximation for the fluid forces.[59] A recent study by Bailie and McFeely[60] has shown this to be true for a panel mounted on a wedge. Their results, using a full unsteady hypersonic theory, which accounts for the change in the steady flowfield, agree very closely with the simpler more approximate theory suggested above.

The principle effect of hypersonic flow on the flutter results is that the flutter dynamic pressure does not increase linearly with Mach number for large M (as predicted by unmodified piston theory) but asymptotes instead to some constant value depending on the wedge angle (or, more generally, the degree of bluntness of the body on which the panel is mounted). The lowest flutter dynamic pressures, however, still occur at low supersonic-transonic freestream Mach number. Therefore, it would appear that, unless the body is extremely blunt and/or transverses a trajectory with maximum freestream dynamic pressure at hypersonic Mach number, the hypersonic regime will not be the most critical with regard to flutter.

The more general question of accounting for local flow conditions at all Mach number remains, however. For some geometries (more complicated than the wedge) it may no longer be possible to treat the mean steady flow over the panel as uniform along the plate length. Whether general analytical or even useful semiempirical methods may be developed remains to be seen. What may well evolve, at least in the near future, are ad hoc tests for the particular configuration to determine the steady flowfield combined with the use of average flow properties in a flutter analysis. More distantly, one can envision the development of method of characteristic procedures or general finite differences schemes.

4.2 Improvements in the Structural Theory and Experiment

While the available structural theories would appear generally satisfactory if one is content to study the elastic (as opposed to inelastic) behavior of the material, there are some aspects of the structural description which deserve further study. In particular, a systematic exploitation of the nonlinear theory is to be recommended for pressure or thermal loaded plates and shells. Also the complexities of sandwich panels, corrugated panels, etc., have yet to be overcome in a quantitative way. Experiments for determination of natural modes and frequencies would be very valuable.

4.3 Improvements in Flutter Theory and Experiment

For flat plates of large length/width ratio, the situation remains unclear. Present methods of analysis are tedious to apply at best and become impractical beyond a certain point. Whether this problem will be alleviated by improved methods of analysis or larger faster computing machines remains to be seen. A very recent and very helpful analysis (type 1) has been presented by Spriggs, Messiter, and Anderson.[61] They have shown the importance of a structural boundary-layer for two-dimensional plates under large tension, i.e., a membrane. Their analysis is mathematically equivalent to the problem of a large a/b plate. Hence, one may interpret their study to say that for such plates one must allow for two length scales. This explains the failure of infinitely long plate analysis which are single scale models. See a forthcoming discussion (Ref. 62) which describes the use of the results for Spriggs et al., for low aspect ratio ($a/b \gg 1$) plates.

For curved plates and shells, the greatest immediate need is for the use of the full linearized potential flow theory in a nonlinear flutter analysis to make a systematic study of the flutter motion over a significant range of variables. One may expect considerable progress in this area in the near future. Additional experiments for curved plates and shells would also be desirable, particularly the former. Finally, it is most desirable that future flutter experiments be carried well into the flutter regime to investigate the limit cycle nature of the flutter oscillation.

5. Design Considerations

Perhaps the most heartening aspect of panel flutter work in recent years has been the advances which have been made in providing experimental and theoretical tools suitable for use in design. A decade ago the subject was notorious for the lack of agreement between theory and experiment and the unreliability of predictive techniques for design. Today, it is fair to say, there is good agreement between theory and experiment for flutter boundaries of flat plates under a variety of applied loads and over a considerable range of Mach number. For curved plates and shells the range of comparison is limited by the limited experimental and theoretical data available. However, the only serious discrepancies which presently exist are for the cylindrial shell under pressurization. Hence, although there is considerable work yet to be done, our confidence level in both theoretical and experimental techniques has increased dramatically in recent years. Indeed, it has reached the stage where most investigators would contend that as far as predictions (by theory or experiment) of flutter boundaries are concerned, these are directly related to our ability to determine the natural vibration modes and frequencies of the panel structure. For pressurized or buckled structure, of course, even this may require a nonlinear structural model. In any event, natural modes are basic to structural behavior and must be incorporated in all dynamical response problems, including e.g., response to boundary-layer noise It should be stated, however, lest we give an over-optimistic view, that the determination of natural modes and frequencies of sandwich plates, corrugated plates, etc., may offer a considerable challenge to the designer.

With the experimental observation that some panels flutter over an extended period of time and fail finally due to fatigue, it has been suggested that panels whose useful life need be relatively short may be designed to flutter, That is, flutter at some amplitude and frequency will correspond to a fatigue life of some duration. If this exceeds the operational life of the panel, then flutter may be permitted at least insofar as structural integrity is concerned. The use of nonlinear flutter analysis and concomitant wind tunnel experiments permit such a design decision to be made on a rational basis. Of course, such a decision requires a high confidence level in our predictive techniques. Experimental verification has yet to be obtained.

It is perhaps worthwhile to compare the situation with regard to panel flutter to that of lifting surface flutter. Two distinctions are usually made in terms of design philosophy. It is often said that 1) the number of parameters which can effect panel flutter is much larger than for lifting surfaces and, 2) the number of different panels in a given design is much larger than the number of lifting surfaces. Hence, it is concluded by some that one cannot expect to predict or describe panel flutter as accurately as lifting surface flutter. It seems to the author that this position has less merit today than ever before. This is basically for two reasons: first, our ability to analyze and measure panel flutter has substantially increased and the capability to analyze a large number of configurations and identify the most critical ones has rapidly improved. Once the basic methods of analysis are available, analyzing several different configurations is a bookkeeping problem admirably handled by modern computers. Secondly, many of the parameters associated with panel flutter are now being recognized as of importance in other physical problems. For example, any variation in panel geometry or loading which is significant for panel flutter will also be important for noise response and sonic fatigue; this includes pressurization, buckling under thermal stress, and even the aerodynamic loading due to the structural motion. Indeed, the lifting surface problem itself has been complicated in recent years by thermal stresses, structural nonlinearities, and boundary layer effects to mention but three. At the risk of appearing heretical it is suggested that lifting surface flutter analysis and experiment could usefully employ some of the techniques developed for the panel flutter problem, particularly with regard to the aforementioned parameters.

In closing, some mention should be made of two documents widely available for design. The first (Ref. 63), prepared by NASA, is very brief and gives a commentary on what they considered to be the essential literature. Recommendations are made for specific types of analysis and/or experiment and margins of safety are briefly discussed along with rules of thumb for identification of those panels most likely to encounter flutter. The basic philosophy is that theory should be used to identify the most critical panels and experiments should be used to establish the flutter boundaries for these panels. No panels should be permitted to undergo destructive flutter. The second and more recent design document (Ref. 64), prepared by Lemley under the sponsorship of the Air Force Flight Dynamics Laboratory, gives simplified criteria in graphical form for most, though not all, of those parameters which may be important for panel flutter design. It assumes very little previous knowledge of the subject on the part of the reader. No panel shall be permitted to flutter. If one may oversimplify a bit, Ref. 64 is useful in determining whether one is likely to have a panel flutter problem while Ref. 63 can help the experienced aeroelastician solve the problem. Actually, Ref. 63 is now rather badly dated and no longer serves its purpose as well as it might. It is to be hoped that it will be revised and up-dated. Neither document describes a method for designing panels for fatigue rather than catastrophic failure, nor inclusion of boundary-layer effects, nor any accurate means for handling loaded (pressurized or buckled) plates.

References

[1] Dowell, E. H., "Nonlinear Theory of the Flutter of Plates and Shells," *Fluid-Solid Interaction Symposium*, edited by J. E. Greenspon, American Society of Mechanical Engineers Winter Meeting, Pittsburgh, Pa., Nov. 1967.

[2] Fung, Y. C., "A Summary of the Theories and Experiments on Panel Flutter," AFOSR TN 60-224, May 1960, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[3] Johns, D. J., "The Present Status of Panel Flutter," Rept. 484, 1961, Advisory Group for Aeronautical Research and Development.

[4] Johns, D. J., "A Survey on Panel Flutter," Advisory Rept. 1, Nov. 1965, Advisory Group for Aeronautical Research and Development.

[5] Dowell, E. H. and Voss, H. M., "Experimental and Theoretical Panel Flutter Studies in the Mach Number Range 1.0 to 5.0," TDR-63-449, Dec. 1963, Aeronautical Systems Division, United States Air Force; also *AIAA Journal*, Vol. 3, No. 12, Dec. 1965, pp. 2292–2304.

[6] Muhlstein, L. Jr., "A Forced-Vibration Technique for Investigation of Panel Flutter," AIAA Paper 66-769, Los Angeles, Calif., 1966.

[7] Dowell, E. H., "Nonlinear Flutter of Curved Plates," *AIAA Journal*, Vol. 7, No. 3, March 1969, pp. 424–431.

[8] Lock, M. H. and Fung, Y. C., "Comparative Experimental and Theoretical Studies of the Flutter of Flat Panels in a Low Supersonic Flow," AFOSR TN 670, May 1961, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[9] Shideler, J. L., Dixon, S. C., and Shore, C. P., "Flutter at Mach 3 Thermally Stressed Panels and Comparison with Theory for Panels with Edge Rotational Restraint," TN D-3498, Aug. 1966, NASA.

[10] Hedgepeth, J. M., "Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds," *Journal of the Aeronautical Sciences*, Vol. 24, No. 8, Aug. 1957, pp. 563–573, and p. 586.

[11] Dugundji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, No. 7, July 1966, pp. 1257–1266.

[12] Cunningham, H. J., "Flutter Analysis of Flat Rectangular Panels Based on Three-Dimensional Supersonic Unsteady Potential Flow," TR R-256, 1967, NASA.

[13] Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate I," *AIAA Journal*, Vol. 4, No. 7, July 1966, pp. 1267–1275.

[14] Fung, Y. C., "The Flutter of a Buckled Plate in a Supersonic Flow," AFOSR TN 55-237, 1955, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[15] Bolotin, V. V., *"Nonconservative Problems of the Theory of Elastic Stability,"* MacMillan Co., New York, 1963, pp. 274–312.

[16] Fralich, R. W., "Postbuckling Effects on the Flutter of Simply Supported Rectangular Panels at Supersonic Speeds," TN D-1615, March 1963, NASA.

[17] Kobayashi, Shigeo, "Flutter of Simply Supported Rectangular Panels in a Supersonic Flow—Two-Dimensional Panel

Flutter, I—Simply Supported Panel, II—Clamped Panel," *Transactions of Japan Society of Aeronautical and Space Sciences*, Vol. 5, 1962, pp. 79–118.

[18] Librescu, L., "Aeroelastic Stability of Orthotropic Heterogeneous Thin Panels in the Vicinity of the Flutter Critical Boundary, Part I," *Journal de Mecanique*, Vol. 4, No. 1, March 1965, pp. 51–76.

[19] Dowell, E. H., "Nonlinear Oscillations of a Fluttering Plate II," *AIAA Journal*, Vol. 5, No. 10, Oct. 1967, pp. 1856–1862.

[20] Dowell, E. H., "Generalized Aerodynamic Forces on a Flexible Plate Undergoing Transient Motion," *Quarterly of Applied Mathematics*, Vol. 24, No. 4, Jan. 1967, pp. 331–338.

[21] Olson, M. D., "Finite Elements Applied to Panel Flutter," *AIAA Journal*, Vol. 5, No. 12, Dec. 1967, pp. 2267–2270.

[22] Dowell, E. H., "Flutter of Infinitely Long Plates and Shells. Part I—Plate," *AIAA Journal*, Vol. 4, No. 8, Aug. 1966, pp. 1370–1377.

[23] Dixon, S. C., "Comparison of Panel Flutter Results from Approximate Aerodynamic Theory with Results from Exact Inviscid Theory and Experiment," TN D-3649, 1966, NASA.

[24] Fung, Y. C., "Flutter of Curved Plates with Edge Compression in a Supersonic Flow," AFOSR TN 57-187, 1957, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.; also "On Two-Dimensional Panel Flutter," *Journal of the Aerospace Sciences*, Vol. 25, No. 3, March 1958, pp. 145–160.

[25] Ventres, C. S. and Dowell, E. H., "Influence of In-plane Edge Support Flexibility on the Nonlinear Flutter of Loaded Plates," *AIAA Structural Dynamics and Aeroelasticity Specialist Conference*, AIAA, New York, 1969.

[26] Houbolt, J. C., "A Study of Several Aero-Thermoelastic Problems of Aircraft Structures in High Speed Flight," Ph.D. thesis, 1958, No. 5, Mitteilungen aus dem Institut für Flugzeugstatik und Leichtbau, Leeman, Zurick.

[27] Schaeffer, H. G. and Heard, W. L., Jr., "Flutter of a Flat Plate Exposed to a Nonlinear Temperature Distribution," *AIAA Journal*, Vol. 3, No. 10, Oct. 1965, pp. 1918–1923.

[28] Voss, H. M., "The Effect of an External Supersonic Flow on the Vibration Characteristics of Thin Cylindrical Shells," *Journal of the Aerospace Sciences*, Vol. 28, No. 12, Dec. 1961, pp. 945–956.

[29] Dowell, E. H., "Nonlinear Flutter of Curved Plates II," *AIAA Journal*, Vol. 8, No. 2, Feb. 1970, pp. 261–263.

[30] Presnell, J. G., Jr. and McKinney, R. L., "Experimental Panel Flutter Results for Some Flat and Curved Titanium Skin Panels at Supersonic Speeds," TN D-1600, Jan. 1963, NASA.

[31] Hess, R. W. and Gibson, F. W., "Experimental Investigation of the Effects of Compressive Stress on the Flutter of a Curved Panel and a Flat Panel at Supersonic Mach Numbers," TN D-1386, Oct. 1962, NASA.

[32] Yates, J. E. and Zeydel, E. F. E., "Flutter of Curved Panels," AFOSR TN 59-163, 1959, Midwest Research Institute, Kansas City, Mo.

[33] Leonard, R. W. and Hedgepeth, J. M., "On Panel Flutter and Divergence of Infinitely Long Unstiffened and Ring-Stiffened Thin-Walled Circular Cylinders," TR 1302, 1957, NASA.

[34] Miles, J. W., "Supersonic Flutter of a Cylindrical Shell, Part I," *Journal of the Aeronautical Sciences*, Vol. 24, No. 2, Feb. 1957, pp. 107–118.

[35] Dowell, E. H., "Flutter of Infinitely Long Plates and Shells: Part II Cylindrical Shell," *AIAA Journal*, Vol. 4, No. 9, Sept. 1966, pp. 1510–1518.

[36] Olson, M. D. and Fung, Y. C., "Comparing Theory and Experiment for the Supersonic Flutter of Cylindrical Shells," *AIAA Journal*, Vol. 5, No. 10, Oct. 1967, pp. 1849–1855.

[37] Evensen, D. A. and Olson, M. D., "Nonlinear Flutter of a Circular Cylindrical Shell in Supersonic Flow," TN D-4265, 1967, NASA.

[38] Carter, L. L. and Stearman, R. O., "Some Aspects of Cylindrical Shell Panel Flutter," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 37–43.

[39] Barr, G. W. and Stearman, R. O., "Aeroelastic Stability Characteristics of Cylindrical Shells Considering Imperfections and Edge Constraint," *AIAA Journal*, Vol. 7, No. 5, May 1969, pp. 912–919.

[40] Anderson, W. J., "Experiments on the Flutter of Flat and Slightly Curved Panels at M = 2.18," AFOSR TN 2996, 1962, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[41] Muhlstein, L., Jr., Gaspers, P. A., Jr., and Riddle, D. W., "An Experimental Study of the Influence of the Turbulent Boundary Layer on Panel Flutter," TN D-4486, 1968, NASA.

[42] Dowell, E. H., "Theoretical-Experimental Correlation of Plate Flutter Boundaries at Low Supersonic Speeds," *AIAA Journal*, Vol. 6, No. 9, Sept. 1969, pp. 1810–1811.

[43] Lemley, C. E., "The Effect of In-Plane Edge Restraint on the Vibration and Flutter of Slight Curved Panels," D.Sc. thesis, 1967, Washington Univ.

[44] Stearman, R. O., Lock, M. H. and Fung, Y. C., "Ames Tests on the Flutter of Cylindrical Shells," Structural Dynamics Rept., SM 62-37, 1962, Graduate Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[45] Olson, M. D. and Fung, Y. C., "Supersonic Flutter of Circular Cylindrical Shells Subjected to Internal Pressure and Axial Compression," *AIAA Journal*, Vol. 4, No. 5, May 1966, pp. 858–864.

[46] Stearman, R. O., "An Experimental Study on the Aeroelastic Stability of Thin Cylindrical Shells at the Lowever Supersonic Mach Numbers," AFOSR 66-2828 and ARL 67-0006, 1966, Midwest Research Institute, Kansas City, Mo.

[47] Dowell, E. H. and Widnall, S. E., "Generalized Aerodynamic Forces on an Oscillating Cylindrical Shell," *Quarterly of Applied Mathematics*, Vol. 24, No. 1, April 1966, pp. 1–17.

[48] Dowell, E. H. and Widnall, S. E., "Generalized Aerodynamic Forces on an Oscillating Cylindrical Shell—Subsonic Flow," *AIAA Journal*, Vol. 4, No. 4, April 1966, pp. 607–610.

[49] Dowell, E. H., "Generalized Aerodynamic Forces on a Flexible Cylindrical Shell Undergoing Transient Motion," *Quarterly of Applied Mathematics*, Vol. 26, No. 3, Oct. 1968, pp. 343–353.

[50] Anderson, W. J., "Oscillatory Pressures in an Idealized Boundary Layer with Application to Cylinder Flutter," *AIAA Journal*, Vol. 4, No. 5, May 1966, pp. 865–872.

[51] Miles, J. W., "On Panel Flutter in the Presence of a Boundary Layer," *Journal of the Aerospace Sciences*, Vol. 26, No. 2, Feb. 1959, pp. 81–93.

[52] McClure, J. D., "On Perturbed Boundary Layer Flows," Rept. 62-2, Fluid Dynamics Research Lab., 1962, Massachusetts Institute of Technology, Cambridge, Mass.

[53] Olson, M. D., "On Comparing Theory and Experiment for the Supersonic Flutter of Circular Cylindrical Shells," AFOSR 66-0944, 1966, Graduate Aeronautical Lab., California Institute of Technology, Pasadena, Calif.

[54] Kurtz, E. F. Jr. and Crandall, S. H., "Computer-Aided Analysis of Hydrodynamic Stability," *Journal of Mathematics and Physics*, Vol. 41, No. 4, Dec. 1962, pp. 264–279.

[55] Gallagher, A. P. and Mercer, A., McD., "On the Behavior of Small Disturbances in Plane Couette Flow," *Journal of Fluid Mechanics*, Vol. 13, Pt. 1, May 1962, pp. 91–100.

[56] Kaplan, R. E., "The Stability of Laminar Incompressible Boundary Layers in the Presence of Compliant Boundaries," TR 116-1, 1964, Aeroelastic and Structures Research Lab., Massachusetts Institute of Technology, Cambridge, Mass.

[57] Zeydel, E. F. E., "Study of the Presence Distribution on Oscillating Panels in Low Supersonic Flow with Turbulent Boundary Layer," NASA CR-691, 1967, Georgia Institute of Technology, Atlanta, Ga.

[58] Dowell, E. H., "Generalized Aerodynamic Forces on a Plate Undergoing Transient Motion in a Shear Flow with an Application to Panel Flutter," AIAA Paper 70–76, New York, 1970.

[59] Shirk, M. H. and Olsen, J. J., "Recent Panel Flutter Research and Applications," Advisory Group for Aeronautical Research and Development Rept. 475, Sept. 1963.

[60] Bailie, J. A. and McFeely, J. E., "Panel Flutter in Hypersonic Flow," *AIAA Journal*, Vol. 6, No. 2, Feb. 1968, pp. 332–337.

[61] Spriggs, J. H., Messiter, A. F., and Anderson, W. J., "Membrane Flutter Paradox—An Explanation by Singular Perturbation Methods," *AIAA Journal*, Vol. 7, No. 9, Sept. 1969, pp. 1704–1709.

[62] Dowell, E. H. and Ventres, C. S., "On the Flutter of Low Aspect Ratio Plates," *AIAA Journal*, to be published.

[63] Lemley, C. E., "Design Criteria for the Prediction of Panel Flutter," TR 67-140, Vols. I and II, Aug. 1968, Air Force Flight Dynamics Lab.

[64] "Panel Flutter," *NASA Space Vehicle Design Criteria*, NASA SP-8004, July 1964.